Suppose you are an instructor in a class. You give an exam to your students, and a particular question has very poor responses. You decide that the question was unfair and should not be counted against anyone who missed it.

There are two easy ways of going about this. You could grade the exam as though everyone got the question correct, or you could omit the question from the exam entirely. Suppose you are concerned about the ramifications of your decision. You have a class policy that no decision on your part should ever negatively affect the grade of a student. This paper seeks to demonstrate that among these two approaches, omitting the question from the exam violates your class policy against negatively impacting a student's grade.

We will need to keep the following in mind:

$$0 < d < w < b \leq t, \tag{1}$$

and

$$w + d = b, \tag{2}$$

where d is the number of points allocated to correctly answering the bad question, w is the "worse" grade of a student who answered the question incorrectly, b is the "better" grade of a student who answered the question correctly, and t is the total number of possible points for the exam.

Let us first examine the case of a student who missed the question. We are asking the question of whether this student's percentage on the exam will be greater if the question is assumed correct or if it is omitted. Let's call the percentages p_1 and p_2.

If the question is omitted, then

$$p_1 = \frac{w}{t - d},$$

and if it is assumed correct, then

$$p_2 = \frac{w + d}{t}.$$

It should be noted that both p_1 and p_2 are greater than the original percentage, so there is no violation of the class policy from the perspective of this student. However, in the interest of curiosity, let's see which choice gives a better percentage? Well, by equations (1) and (2),

$$w + d = b \leq t,$$

$$\therefore d(w + d) \leq dt,$$

$$\therefore 0 \leq dt - d(w + d),$$

$$\therefore wt \leq wt + (dt - d(w + d)) = wt + dt - dw - d^2,$$

$$\therefore wt \leq (w+d)(t-d),$$

$$\therefore \frac{w}{t-d} \leq \frac{w+d}{t},$$

$$\therefore p_1 \leq p_2.$$

This proves that in the case of a student who missed the question, their average will be better if they are given credit for the question rather than the question being omitted.

Now we shall examine the case of the student who first answered the question correctly. Let's now let p_0 be their grade before the question is omitted and p_1 be their grade after it is omitted. Then

$$p_0 = \frac{b}{t},$$

and

$$p_1 = \frac{b-d}{t-d}.$$

By equation (1),

$$b \leq t,$$

$$\therefore bd \leq td,$$

$$\therefore -bd \geq -td,$$

$$\therefore bt - bd \geq bt - td,$$

$$\therefore b(t-d) \geq t(b-d),$$

$$\therefore \frac{b}{t} \geq \frac{b-d}{t-d},$$

$$\therefore p_0 \geq p_1.$$

So, in all cases except when the student got a perfect score, their grade is negatively influenced by omitting the question from the exam. This directly violates the class policy against negatively affecting a student's grade by the instructor's decision.